

on the attenuation coefficient is not as marked. The results confirm that the change in the propagation coefficient is due to the longitudinal magnetoresistance effect. The results also show that the longitudinal magnetoresistance effect at 30 GHz is of the same order as the dc value. This was expected from previous investigations [8], [9] which showed the frequency independence of transverse magnetoresistance effects.

#### EXPERIMENTAL DETAILS

The experimental results were obtained using a microwave transmission bridge operating at 30 GHz and carefully adjusted to avoid internal reflections within the bridge. The semiconductor samples were cut from a single crystal block of germanium using a diamond saw and then lapped to the required dimensions. The internal dimensions of the waveguide are  $7.112 \times 3.556$  mm  $\pm$  0.02 mm and the samples were lapped to  $7.05 \times 3.53$  mm to enable them to be inserted and removed from the waveguide without chipping or cracking. The samples were lapped to size on fine (1000) carborundum paper and the front and back faces were then polished with a fine aluminum oxide powder. The samples were then quickly etched in an acid solution and washed in acetone and distilled water.

The effect of the air gap between the sample and the waveguide walls has been discussed in [5] and correction terms are given. For this case the correction terms are small and were reduced further by coating the sides of the sample in contact with the waveguide walls with a highly conducting silver epoxy. However, the air gap still had a slight but noticeable effect on the phase coefficient. This effect is larger when the air gap is distributed equally at both the broad walls of the guide rather than when one side of the sample is in complete contact with the broad wall.

DC measurements of the conductivity and magnetoresistivity were made with rectangular specimens suitably etched to remove any higher conductivity surface layers and leads were soldered to the samples using Sn/Sb solder.

Since single crystal samples were used, it was necessary to ensure that the current and magnetic-field directions retained the same orientation with respect to the crystal axes for both the dc and microwave measurements. The direction of microwave propagation was always taken along the  $\langle 111 \rangle$  crystal axis.

#### CONCLUSION

The effect of a steady magnetic field on microwave propagation through a fully filled semiconductor-loaded waveguide has been analyzed. When the magnetic field is perpendicular to the electric field of the incident  $TE_{10}$  mode, the propagation can be explained in terms of the distortion of the  $TE_{10}$  mode by the Hall effect. The simple theory of spherical constant-energy surfaces is shown to give good results in this case. Although the transverse magnetoresistance effect is implicit in this method, this effect itself does not adequately account for the magnetic-field dependency of the propagation coefficient. When the magnetic field is parallel to the incident electric field, the  $TE_{10}$  mode is the propagating mode and the effect of the magnetic field on the propagation coefficient is explained by longitudinal magnetoresistance effects. The results indicate that magnetoresistance is frequency independent for frequencies at least as high as 30 GHz.

The relaxation time  $\tau$  has also been shown to affect the propagation coefficient at this frequency, although it would be difficult to estimate the relaxation time from these measurements. To enable  $\tau$  to be measured in this way the conductivity and dielectric

constant would need to be known very accurately. Experimental problems, such as sample inhomogeneity and the limited accuracy of commercially available microwave components, would also need to be overcome. However, this method of measurement provides a simple means of measuring the Hall mobility of semiconductors at microwave frequencies.

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#### Propagation Along a Braided Coaxial Cable Located Close to a Tunnel Wall

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**Abstract**—A previous development is extended to permit attenuation calculations when a braided cable is located close to a tunnel wall. This is an important case in mine communications utilizing leaky feeders. Numerical results are presented to illustrate the effects of numerous parameters on mode attenuation. A principal finding is that the attenuation rate for the bifilar mode is hardly affected at all by the finite conductivity of the wall. On the other hand, the monofilar mode suffers a very high attenuation when the cable approaches the wall.

#### INTRODUCTION

The leaky-feeder technique is now being developed for communication in mines [1]. In this method, referred to as continuous-access guided communications (CAGC), the signals are guided by some type of transmission line. The energy is coupled into or out of the channel by antennas in the vicinity of the transmission line which may be a coaxial cable [2] or a twin-wire line [3], [4].

We have previously derived a mode equation for a braided coaxial cable within a circular tunnel and we presented some numerical results [5]. However, that mode equation is very poorly convergent when the cable is located close to the tunnel wall. Unfortunately, it is precisely this case which is of most practical interest for communication in coal mines where it is generally necessary to lay the cable close to the wall [6]. In this

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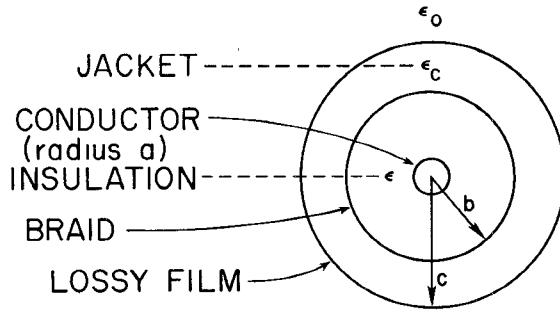


Fig. 1. The braided coaxial cable.

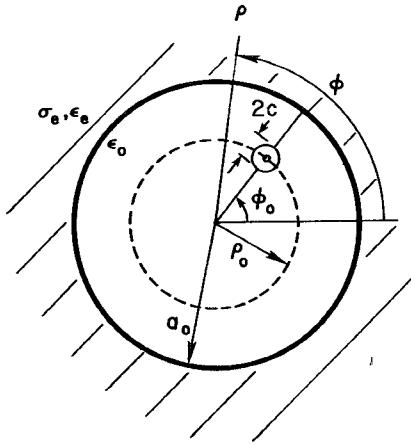


Fig. 2. The geometry of the cable in a circular tunnel.

short paper, we derive a rapidly convergent mode equation which is used to obtain numerical results for the attenuation rates of the dominant modes when the cable is located close to a circular tunnel wall.

#### MODE EQUATION

The geometry of the braided coaxial cable is shown in Fig. 1. The center conductor of radius  $a$  is assumed to have a very high but finite conductivity  $\sigma_w$ . The insulation of radius  $b$  is a lossless dielectric of permittivity  $\epsilon$ . The metal braid of radius  $b$  is represented by a transfer impedance  $Z_T$  which is given by [2]

$$Z_T = i\omega L_T \quad (1)$$

where  $L_T$  is the transfer inductance and  $\exp(i\omega t)$  time dependence is assumed. The coating of radius  $c$  is a lossless dielectric of permittivity  $\epsilon_c$ . A thin lossy film of radius  $c$  is characterized by a transfer impedance  $Z_L$  which is given by [5]

$$Z_L = (2\pi c\sigma d)^{-1} \quad (2)$$

where  $\sigma d$  is the conductivity-thickness product of the film. Rawat and Beal [7] indicate that the presence of such lossy films should be expected in realistic mine environments due to moisture and/or dust accumulation.

The idealized circular-tunnel geometry and the cylindrical coordinate system  $(\rho, \phi, z)$  are shown in Fig. 2. The air-filled tunnel has a radius  $a_0$  and is bounded by a lossy dielectric of conductivity  $\sigma_e$  and permittivity  $\epsilon_e$ . Free-space permeability  $\mu_0$  is assumed everywhere. The braided cable is located within the tunnel at  $(\rho_0, \phi_0)$ .

We assume that the individual modes of the structure vary

in the  $z$  direction as  $\exp(-\Gamma z)$  for a time factor  $\exp(i\omega t)$  and the objective is to calculate the values of the propagation constant  $\Gamma$ . By matching the tangential-field components at the tunnel boundary ( $\rho = a_0$ ) and applying an impedance condition at the edge of the cable ( $\rho = \rho_0 + c$ ,  $\phi = \phi_0$ ), the following mode equation has been derived [5], [8]:

$$\frac{i\omega\mu_0v^2}{2\pi\gamma_0^2} [K_0(vc) - S] - Z(\Gamma) = 0 \quad (3)$$

where  $v = (\gamma_0^2 - \Gamma^2)^{1/2}$ ,  $\gamma_0^2 = \omega^2\mu_0\epsilon_0$ , and  $K_0$  is the modified Bessel function of the second kind. The summation of cylindrical harmonics  $S$  represents the effect of the tunnel boundary and is given by [8]

$$S = \sum_{m=0,1,2,\dots}^{\infty} T_m \quad (4)$$

where

$$T_m = \epsilon_m R_m \frac{K_m(va_0)}{I_m(va_0)} I_m(v\rho_0) I_m(v(\rho_0 + c))$$

$$\epsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m \neq 0 \end{cases}$$

$$R_m = \frac{[(\gamma_0/v)K_m'(va_0)/K_m(va_0)] + Y_m\eta_0 + \delta_m\eta_0}{[(\gamma_0/v)I_m'(va_0)/I_m(va_0)] + Y_m\eta_0 + \delta_m\eta_0}$$

$$Y_m = \left( \frac{i\gamma_e^2}{u\mu_0\omega} \right) \frac{K_m'(ua_0)}{K_m(ua_0)} \quad \eta_0 = (\mu_0/\epsilon_0)^{1/2}$$

$$\delta_m\eta_0 = \frac{(im\Gamma/a_0)^2(v^{-2} - u^{-2})^2}{[(\gamma_0/v)I_m'(va_0)/I_m(va_0)] + Z_m/\eta_0}$$

$$Z_m = - \left( \frac{i\omega\mu_0}{u} \right) \frac{K_m'(ua_0)}{K_m(ua_0)}$$

$$\gamma_e^2 = i\omega\mu_0(\sigma_e + i\omega\epsilon_e) \quad u = (\gamma_e^2 - \Gamma^2)^{1/2}.$$

$I_m$  and  $K_m$  are modified Bessel functions of the first and second kind, and the prime indicates differentiation with respect to the argument. The series impedance per unit length of the cable  $Z(\Gamma)$  has the following form, provided that the cable radius  $c$  is electrically small [5]:

$$Z(\Gamma) = \frac{Z_L(Z_c + Z_b)}{Z_L + Z_c + Z_b} \quad (5)$$

$$Z_b = \frac{Z_T(Z' + Z_i)}{Z_T + Z' + Z_i}$$

where

$$Z' = \frac{\gamma^2 - \Gamma^2}{2\pi i\omega\epsilon} \ln(b/a), \quad \gamma^2 = -\omega^2\mu_0\epsilon$$

$$Z_c = \frac{\gamma_c^2 - \Gamma^2}{2\pi i\omega\epsilon_c} \ln(c/b), \quad \gamma_c^2 = -\omega^2\mu_0\epsilon_c$$

$$Z_i = \frac{(i\omega\mu_0)^{1/2}}{(2\pi c a \sigma_w)^{1/2}} \frac{I_0(\gamma_w a)}{I_1(\gamma_w a)}, \quad \gamma_w = (i\omega\mu_0\sigma_w)^{1/2}$$

and  $|\gamma_w|$  is assumed much larger than  $|\Gamma|$ .

#### ASYMPTOTIC BEHAVIOR

Although some approximations to the mode equation (3) are possible under certain limiting conditions, a numerical evaluation of (3) is generally required. Numerical results for the propagation

constant  $\Gamma$  have been obtained using a modification of Newton's method [5], but convergence problems inhibit a direct evaluation of  $S$  as given by (4) when  $\rho_0/a_0$  is nearly equal to unity. Here we develop an efficient method to treat this important case where the cable is near the wall.

The first step is to examine the asymptotic behavior of the  $m$ th term  $T_m$  in (4) as  $m$  becomes large. The first terms of the required uniform asymptotic expansions [9] are

$$\begin{aligned} I_v(vz) &\sim \frac{1}{(2\pi v)^{1/2}} \frac{\exp(v\eta)}{(1+z^2)^{1/4}} \\ K_v(vz) &\sim \left(\frac{\pi}{2v}\right)^{1/4} \frac{\exp(-v\eta)}{(1+z^2)^{1/4}} \\ I'_v(vz) &\sim \frac{1}{(2\pi v)^{1/2}} \frac{(1+z^2)^{1/4} \exp(v\eta)}{z} \\ K'_v(vz) &\sim \frac{1}{(2\pi v)^{1/2}} \frac{(1+z^2)^{1/4} \exp(-v\eta)}{z} \end{aligned} \quad (6)$$

where

$$\eta = (1+z^2)^{1/2} + \ln \left[ \frac{z}{1+(1+z^2)^{1/2}} \right].$$

If we let  $m = v$  and  $x = vz$  and if  $m \gg |x|$ , the following can be derived from (6):

$$\begin{aligned} I_m(x) &\sim \frac{1}{(2\pi m)^{1/2}} \left(\frac{e}{2m}\right)^m x^m \\ K_m(x) &\sim \left(\frac{\pi}{2m}\right)^{1/2} \left(\frac{e}{2m}\right)^{-m} x^{-m} \\ \frac{I'_m(x)}{I_m(x)} &\sim \frac{m}{x} \\ \frac{K'_m(x)}{K_m(x)} &\sim -\frac{m}{x}. \end{aligned} \quad (7)$$

The expressions in (7) also reveal the problem that  $I_m$  can become too small and  $K_m$  can become too large for computer storage as  $m$  becomes large. However, by substituting (7) into (4) the following simple asymptotic expression is obtained for  $T_m$ :

$$T_m \sim \frac{R^a}{m} \left[ \frac{\rho_0(\rho_0+c)}{a_0^2} \right]^m = T_m^a \quad (8)$$

where  $R^a$  is the asymptotic expression for  $R_m$  and is independent of  $m$ . The explicit expression for  $R^a$  is obtained by dividing the numerator and denominator of  $R_m$  by  $m$  and substituting the asymptotic expressions from (7)

$$R^a = \frac{-\frac{\gamma_0}{v^2 a_0} + \frac{Y_m^a \eta_0}{m} + \frac{\delta_m^a \eta_0}{m}}{\frac{\gamma_0}{v^2 a_0} + \frac{Y_m^a \eta_0}{m} + \frac{\delta_m^a \eta_0}{m}} \quad (9)$$

where

$$\begin{aligned} \frac{Y_m^a \eta_0}{m} &= \frac{-i\gamma_e^2 \eta_0}{\omega \mu_0 u^2 a_0} \\ \frac{\delta_m^a \eta_0}{m} &= \frac{(i\Gamma/a_0)^2 (v^{-2} - u^{-2})^2}{\frac{\gamma_0}{v^2 a_0} + \frac{Z_m^a}{\eta_0 m}} \end{aligned}$$

and

$$\frac{Z_m^a}{\eta_0 m} = \frac{i\omega \mu_0}{u^2 a_0 \eta_0}.$$

The poor convergence of (4) for  $\rho_0/a_0$  close to unity is apparent from the expression for  $T_m^a$  in (8). However, the following summation formula [10] may be used:

$$\sum_{m=1}^{\infty} \frac{r^m}{m} = -\ln(1-r). \quad (10)$$

Thus, from (8) and (10), we can write

$$\sum_{m=1}^{\infty} T_m^a = -R^a \ln \left[ 1 - \frac{\rho_0(\rho_0+c)}{a_0^2} \right]. \quad (11)$$

By subtracting (11) from (4), the following rapidly convergent form is obtained for  $S$ :

$$\begin{aligned} S &= -R^a \ln \left[ 1 - \frac{\rho_0(\rho_0+c)}{a_0^2} \right] \\ &\quad + T_0 + \sum_{m=1}^{\infty} (T_m - T_m^a). \end{aligned} \quad (12)$$

This form is used in the mode equation (3) to obtain numerical results.

If the tunnel becomes very large electrically, the arguments of the Bessel functions ( $va_0$  and  $ua_0$ ) can be quite large and (12) may not converge rapidly. However, computation time could still be decreased by employing asymptotic expansions which are valid for large order and large argument. These are

$$\begin{aligned} \frac{I'_m(x)}{I_m(x)} &\sim \frac{m}{x} \left(1 - \frac{x^2}{m^2}\right)^{1/2} \\ \frac{K'_m(x)}{K_m(x)} &\sim \frac{-m}{x} \left(1 - \frac{x^2}{m^2}\right)^{1/2} \end{aligned} \quad (13)$$

but they are not valid when  $x$  is near  $m$ . Wait [11] has examined such approximations in a study of whispering-gallery modes in electrically large cylinders.

## NUMERICAL RESULTS

Using the mode equation (3) along with the rapidly convergent form of  $S$  in (12), numerical results for the propagation constant  $\Gamma$  were obtained for both the monofilar and bifilar modes. The energy in the bifilar mode is concentrated primarily within the insulation, and the solution is found in the neighborhood  $\Gamma \approx \gamma$ , where  $\gamma$  is the propagation constant of the insulation. For the monofilar mode, the forward current is carried by the cable and the return current is carried by the tunnel wall. Consequently, the solution is found in the neighborhood  $\Gamma \approx \gamma_0$ , where  $\gamma_0$  is the free-space propagation constant. In either case the attenuation rate  $\alpha$  is given by

$$\alpha = \text{Re}(\Gamma)(\text{Np/m}) = 8.686 \times 10^3 \text{ Re}(\Gamma)(\text{dB/km}). \quad (14)$$

In all cases the tunnel radius  $a_0$  was taken as 2 m, and the following cable parameters were used:  $a = 1.5$  mm,  $b = 10$  mm,  $c = 11.5$  mm,  $\sigma_w = 5.7 \times 10^7$  mho/m,  $\epsilon/\epsilon_0 = 2.5$ ,  $\epsilon_c/\epsilon_0 = 3.0$ , and  $\sigma d = 10^{-3}$  mho. All figures cover the frequency range from 1 to 20 MHz. For higher frequencies, (12) is no longer rapidly convergent.

Figs. 3-5 show attenuation rates for the bifilar mode for wall

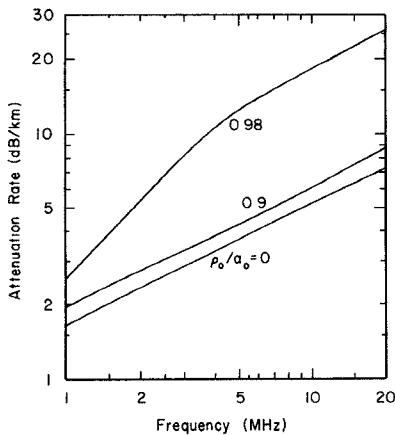


Fig. 3. The effect of cable position on the attenuation rate of the bifilar mode. The cable parameters correspond to the FONT cable. (Parameters:  $L_T = 40$  nH/m,  $a = 1.5$  mm,  $b = 10$  mm,  $c = 11.5$  mm,  $a_0 = 2$  m,  $\epsilon_e/\epsilon_0 = 10$ ,  $\sigma_e = 10^{-3}$  mho/m,  $\sigma_w = 5.7 \times 10^7$  mho/m,  $\sigma_d = 10^{-3}$  mho,  $\epsilon_c/\epsilon_0 = 2.5$ ,  $\epsilon_c/\epsilon_0 = 3.0$ .)

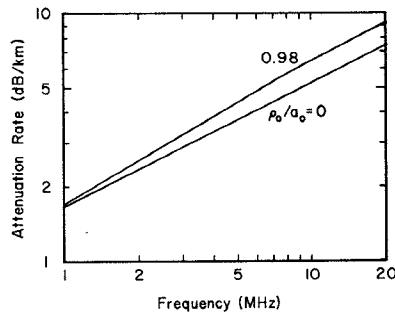


Fig. 4. The attenuation rate of the bifilar mode for a reduced value of  $L_T$ . (Parameters as in Fig. 3, but  $L_T = 10$  nH/m.)

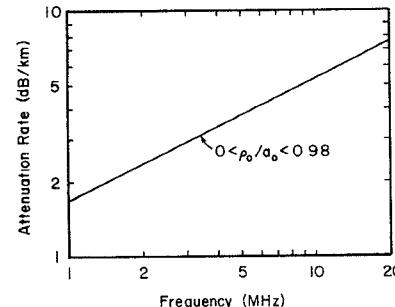


Fig. 5. The attenuation rate of the bifilar mode for a further reduced value of  $L_T$ . (Parameters as in Fig. 3, but  $L_T = 2$  nH/m.)

constants,  $\sigma_e = 10^{-3}$  mho/m and  $\epsilon_e/\epsilon_0 = 10$ . In Fig. 3,  $L_T$  is taken to be 40 nH/m which corresponds to the very high transfer inductance of the FONT cable developed by Fontaine *et al.* [2]. For  $\rho_0/a_0 = 0.98$ , the cable center is only 4 cm from the wall, and the attenuation rate is increased significantly. The optimum frequency for this cable has been claimed to be approximately 7 MHz [2]. Most coaxial cables possess a much lower value of  $L_T$  [12], and Fig. 4 shows the same case with  $L_T$  reduced to 10 nH/m. Fig. 5 shows the same case with  $L_T$  reduced even further to 2 nH/m, and in this case the tunnel wall has essentially no effect on the attenuation rate. Fig. 6 shows the rather complicated effect of wall conductivity on the attenuation rate of the

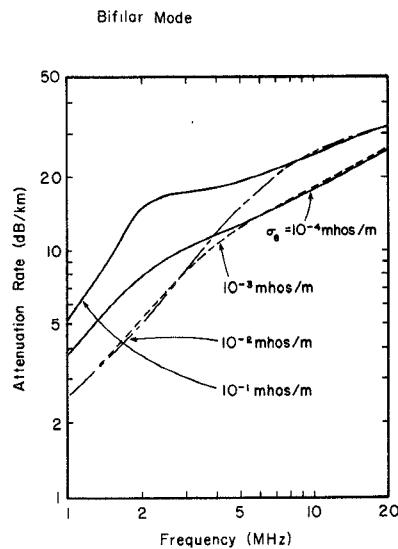


Fig. 6. The effect of tunnel-wall conductivity on the attenuation rate of the bifilar mode. (Parameters:  $a_0 = 2$  m,  $\rho_0/a_0 = 0.98$ ,  $\epsilon_e/\epsilon_0 = 10$ ,  $L_T = 40$  nH/m,  $a = 1.5$  mm,  $b = 10$  mm,  $c = 11.5$  mm,  $\sigma_d = 10^{-3}$  mho,  $\sigma_w = 5.7 \times 10^7$  mho/m,  $\epsilon_c/\epsilon_0 = 2.5$ ,  $\epsilon_c/\epsilon_0 = 3.0$ .)

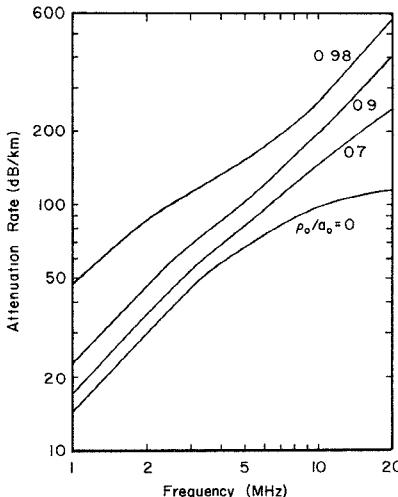


Fig. 7. The effect of cable position on the attenuation rate of the monofilar mode. (Parameters as in Fig. 3.)

bifilar mode for  $\rho_0/a_0 = 0.98$  and  $L_T = 40$  nH/m. For smaller values of  $\rho_0$  and/or  $L_T$ , the wall conductivity has a lesser effect.

Fig. 7 shows the effect of cable position on the attenuation rate of the monofilar mode. Note that the attenuation rate is in general an order of magnitude higher than that of the bifilar mode. Even for the increased wall conductivity ( $\sigma_e = 10^{-1}$  mho/m) shown in Fig. 8, the attenuation rate is still quite high.

#### CONCLUDING REMARKS

A method has been developed for treating the important practical case of the cable close to the tunnel wall. It is found that the tunnel wall has little effect on the attenuation rate of the bifilar mode unless the cable has a very large transfer inductance and is located close to the wall as shown in Fig. 3. The monofilar mode has a high attenuation rate for most cases of interest and is probably of use only if some mode conversion exists between the monofilar and bifilar modes.

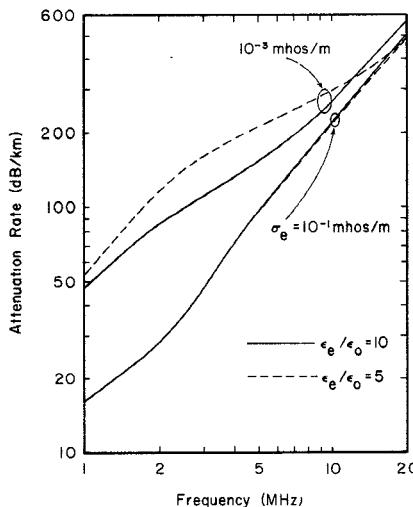


Fig. 8. The effect of tunnel-wall conductivity and permittivity on the attenuation rate of the monofilar mode. (Parameters as in Fig. 6 except for indicated values of  $\epsilon_e/\epsilon_0$  and  $\sigma_e$ .)

An important related area for further work is the excitation (and reception) of the monofilar and particularly the bifilar mode. Quantitative knowledge is required for a total calculation of system loss and communication range. Also the use of higher frequencies with cables close to the wall merits some attention even though higher attenuation rates can be expected.

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## Letters

### Reflection Coefficient of Unequal Displaced Rectangular Waveguides

RALPH LEVY

**Abstract**—The IEC has suggested that maximum allowable displacements of waveguide flanges should not cause the inherent return loss due to waveguide tolerances to degrade more than 1 dB. Calculations on displaced unequal waveguides at their extreme tolerances show that this leads to a maximum allowable displacement of 0.0175 of the broad ( $= a$ ) dimension for a waveguide tolerance of  $\pm a/500$ . The worst return

loss under these conditions is approximately -41 dB. However, it is suggested that this maximum allowable displacement is based on a statistically remote worse case condition, and relaxation to a value of 0.021a would be more realistic.

#### INTRODUCTION

The question of how to specify tolerances on dimensions of locating holes and bolt diameters of rectangular-waveguide flanges, which determine maximum waveguide misalignment, has been under consideration by the International Electro-technical Commission (IEC) Sub-Committee 46B for several years. At their last meeting in Bucharest in 1974 it was proposed that the maximum allowable displacement at a junction of two waveguides shall be such that the degradation of return loss